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LETTER TO THE EDITOR

Thermal crossover effects resulting from dilution-induced magnon critical dynamics

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Abstract. The dynamics of magnons in Heisenberg magnets becomes anomalous when the magnet is diluted to concentrations p near the percolation threshold p_c, where the percolation network becomes self-similar. The resulting crossover in dispersion relation and density of states from hydrodynamic to critical forms affects static thermal properties, such as magnetisation, influenced by excitation of magnons. It is shown that for low temperature T and $\delta p = (p - p_c)$ small the magnetisation of diluted ferromagnets is

$$M(p, T) = P(p) - \delta p^{\nu d} f(\delta p^{\nu z} J / k_B T)$$

where $P(p) \propto \delta p^{\beta}$ is the percolation probability, ν the percolation correlation length exponent, z the dynamic critical exponent, d the Euclidean dimensionality ($d > 2$) and J the exchange constant. The different asymptotic forms of the scaling function $f(y)$ for $y \ll 1$ and $y \gg 1$ imply a crossover in magnetisation decrease $M(p, 0) - M(p, T)$ from standard $T^{d/2}$ (hydrodynamic) behaviour (with non-trivial δp-dependent coefficient because of mode softening) for $T < T^*(p)$ to a form proportional to $T \delta p^{(d-2)\nu}$ for $T > T^*(p)$, where $T^*(p)$ is a crossover temperature proportional to $\delta p^{\nu z}$. This crossover is additional to the standard percolation-thermal crossover which is also present, e.g., in the form of the transition temperature $T_c(p)$ ($\propto \delta p^{(z-d)\nu+\beta}$) resulting from $M(p, T_c(p)) = 0$.

Dynamical processes become critical on self-similar structures, e.g. random or non-random fractals (see, e.g., [1, 2]). One of the most important real cases is in dilute-lattice-based systems, particularly dilute localised magnets at the percolation threshold [2, 3]. This criticality in dynamics will have an effect on those static thermal properties related directly to the excitation of the 'critical modes', e.g. the thermal quantities such as magnetisation and specific heat resulting from excitation of spin waves.

This letter will show that the hydrodynamic-to-critical crossover in the dynamics of Heisenberg spins on the percolation network causes a crossover in the magnetisation behaviour between two different power law dependences on temperature and concentration (the powers involving Euclidean, fractal [4] and fracton [5] dimensions or related critical exponents, and the percolation correlation length exponent). This crossover is different from the usual percolation-thermal crossover [3], which is also present in the theory and evident in the resulting dependence of transition temperature on concentration, which is in agreement with the form obtained previously by static arguments [6, 7].

In dilute magnets at concentrations p near the percolation threshold p_c, the correlation length ξ diverges according to [3]

$$\xi / a \sim (p - p_c)^{-\nu} \tag{1}$$

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where a is the lattice constant. The divergence in ξ induces a crossover in the dynamics of spin waves [2] of wavevector k from hydrodynamic behaviour for $k\xi < 1$ to critical behaviour for $k\xi > 1$. We consider the effects of this crossover on the magnetisation of site-diluted Heisenberg ferromagnets.

The (spontaneous) magnetisation arises from spins on the infinite cluster which is present for $p > p_c$. At zero temperature ($T = 0$), the spins in the infinite cluster are aligned so the magnetisation per site in a system of N sites is

$$\frac{M(p,0)}{N} = P(p) \propto (p - p_c)^{\beta} \tag{2}$$

where $P(p)$ is the percolation probability (the probability that an occupied site is part of the infinite cluster) and β is its associated critical exponent [8].

At low finite temperatures excitation of magnons reduces the magnetisation to [9]

$$\frac{M(p,0)}{N} = P(p) - \frac{1}{N}\sum_{k} n(p,k) \tag{3}$$

where $n(p,k) = [\exp(\beta\omega_k) - 1]^{-1}$ is the number of excited spin waves with frequency ω_k on the (configurationally averaged) infinite cluster, and $\beta = 1/k_B T$. k is the inverse characteristic length of the spin excitations (a wavelength in the hydrodynamic regime and a localisation length in the critical regime—see below).

In the following we shall consider low temperatures (so that the 'wavevectors' occurring in (3) are small) and concentrations near p_c where, by (1), ξ is large. In this situation the dynamic scaling hypothesis [10] applies and then the frequency ω_k and density of states have the following different forms in the hydrodynamic and critical regimes [2, 11-14].

(i) Hydrodynamic regime ($k\xi < 1$). The dispersion relation is

$$\omega_k = Dk^2 \qquad D = D_0(\xi/a)^{(2-z)}. \tag{4}$$

The spin wave stiffness, D, is reduced from its pure value ($D_0 = Ja^2$, where J is the exchange constant) by the dependence on correlation length, which gives rise to mode softening as $p \to p_c$ (ξ increasing) since z is greater than 2. z is the dynamic exponent characterising the dynamics in the critical regime (see (ii) below). (In the analogous case of anomalous diffusion [14] z would be the exponent normally written as $(2+\theta)$). The number of magnons with wavevectors in the range $\boldsymbol{k}$ to $\boldsymbol{k} + \mathrm{d}\boldsymbol{k}$ is

$$\rho(\boldsymbol{k})\,\mathrm{d}^d k = Na^d(2\pi)^{-d}\,\mathrm{d}^d k \tag{5}$$

where d is the Euclidean dimension.

(ii) Critical regime ($k\xi > 1$). The dispersion relation now takes the critical form, with dynamic exponent z, where k is now the inverse characteristic length of the localised fracton modes [5, 15]:

$$\omega_k = ck^z \qquad c = D_0(a/\pi)^{(z-2)} \tag{6}$$

and the number of fractons is

$$\rho(\boldsymbol{k})\,\mathrm{d}^{d_f} k = NP(p)a^{d_f}(2\pi)^{-d_f}\,\mathrm{d}^{d_f} k \tag{7}$$

where d_f is the fractal dimension of the infinite cluster, differing from the Euclidean dimension by the 'anomalous dimension' β/ν (length scaling dimension) of $P(p)$:

$$d_f = d - \beta/\nu. \tag{8}$$

The coefficients in (4) and (6) are related by the equivalence of the two forms in the crossover region $k\xi \sim 1$. Equations (5) and (7) count the modes on the infinite cluster: (5) arises from direct mode counting which applies in the hydrodynamic regime, while in (7) the concentration-dependent factor is required to give the correct total number of modes up to a ξ-independent cutoff (minimum length), the two forms connecting smoothly at the crossover [16].

The spin wave sum in (3) can now be split into the two parts corresponding to the sums over wavevectors less than and greater than $1/\xi$, i.e. to the contributions from hydrodynamic and critical regimes. The corresponding dispersion relations and densities of states can then be used to write the two sums as integrals with respect to k (from 0 to $1/\xi$ and from $1/\xi$ to order π/a, respectively) of known functions of k. Changing the integration variable from k to $x = \beta\omega_k$ (which of course is related to k in a different way in the two integrals) leads to, using also (1), (2) and (8),

$$\frac{M(p, T)}{N} = P(p) - A(\xi/a)^{-d} y^{-d/2} I_{d/2}(y, 0) - B(\xi/a)^{-d} y^{-d_f/z} I_{d_f/z}(c_1 \beta J, y). \tag{9}$$

In this expression

$$I_\alpha(\gamma, \delta) \equiv \int_\delta^\gamma \frac{x^{(\alpha-1)}}{e^x - 1}\, dx \tag{10}$$

$y \equiv \beta J(a/\xi)^z$, c_1 is a constant whose value will turn out to be irrelevant, and $A = 2^{-1}(2\pi)^{-d} A_d$ and $B = z^{-1}(2\pi)^{-d_f} A_{d_f}$ where A_d and A_{d_f} are the areas of d- and d_f-dimensional unit hyperspheres, respectively. As is usual in spin wave calculations an integral (in this case $I_{d/2}$) becomes divergent for dimensionality $d \leq 2$. Our analysis therefore only applies for $d > 2$. Also, as could be expected, the spectral dimensionality [5] $d_s = d_f/z$ appears in (9).

Because the analysis is being carried out for low temperatures, the limit $c_1 \beta J$ can be replaced by infinity. The integrals are thus only functions of the variable y, and (9) takes the form

$$\frac{M(p, T)}{N} = P(p) - (p - p_c)^{\nu d} f(y) \tag{11}$$

in which $y = \beta J(a/\xi)^z$ plays the role of a crossover variable. As will be discussed subsequently this characterises a new crossover, induced by the dynamic crossover, which shows up in addition to the usual percolation-thermal crossover. By asymptotic evaluation of the integrals it is not difficult to show that

$$\begin{aligned} f(y) &\propto y^{-1} && y \ll 1 \\ &\propto y^{-d/2} && y \gg 1. \end{aligned} \tag{12}$$

The result for $y \ll 1$ involves contributions from both hydrodynamic and critical spin waves while that for $y \gg 1$ comes entirely from the hydrodynamic regime.

Before proceeding, we now estimate the transition temperature $T_c(p)$ by finding where the magnetisation vanishes ($M(p, T_c(p)) = 0$). Since in general $\nu d > \beta$ this occurs where $f(y)$ is large, i.e. for small y (using (12)) which leads to the phase boundary

$$k_B T_c(p)/J \sim (p - p_c)^{(z-d)\nu+\beta}. \tag{13}$$

Because of the Einstein relation [2, 13] the critical exponent ζ characterising the vanishing of the percolation conductivity at p_c satisfies $\zeta = (z-d)\nu + \beta$; the result (13) for the dependence of transition temperature on concentration near the threshold is therefore in agreement with that [6, 7] obtained by purely static arguments, which yield ζ as the percolation-thermal crossover exponent.

The crossover between the two asymptotic behaviours in (12) occurs where $y \sim 1$, i.e. at the crossover line

$$k_B T/J \sim (p-p_c)^{\nu z}. \tag{14}$$

This is therefore a different crossover (dynamic in origin) from the usual percolation-thermal crossover involved in (13). Because $d\nu > \beta$, the new crossover is at a lower temperature than the critical temperature given by (13), so the crossover line (14) occurs in the ordered region of the phase diagram (which is necessary for the crossover to actually occur). The phase diagram is illustrated in figure 1. The full curve represents the phase boundary (13) between the paramagnetic phase 1 and the ordered phase. The ordered phase is divided by the crossover curve (14), shown broken, into two regions, 2 and 3, in which the two different magnetisation behaviours resulting from inserting (12) into (11) occur.

In region (2) ($y < 1$), where both hydrodynamic and critical spin waves contribute,

$$\frac{M(p,T)}{N} = P(p) - A_0(p-p_c)^{(d-z)\nu}\frac{k_B T}{J} \qquad \left(\frac{k_B T}{J} > (p-p_c)^{\nu z}\right). \tag{15}$$

This expression vanishes at the critical line (13). A_0 is the dimensionless constant $A_0 = (d/2-1)^{-1}A + (1-d_f/z)^{-1}B$.

In region 3 ($y > 1$), in which the magnetisation decrease is from only hydrodynamic spin waves,

$$\frac{M(p,T)}{N} = P(p) - B_0(p-p_c)^{(2-z)\nu d/2}\left(\frac{k_B T}{J}\right)^{d/2} \qquad \left(\frac{k_B T}{J} < (p-p_c)^{\nu z}\right). \tag{16}$$

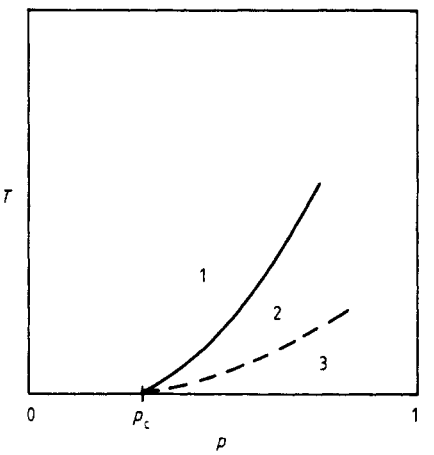

Figure 1. Phase diagram of dilute Heisenberg ferromagnet at low temperature T and concentration p near percolation threshold p_c. The full curve is the phase boundary (13) dividing paramagnetic phase 1 and ferromagnetic phase. The broken crossover curve (14) divides the ferromagnetic phase into two regions, 2 and 3, in which the magnetisation behaviours (15) and (16) respectively apply.

The constant B_0 is $B_0 = AI_{d/2}(\infty, 0)$. In (16) the power of the temperature is the standard one [9], because the wavevector dependencies in (4) and (5) are not anomalous. However a non-trivial power of $(p - p_c)$ occurs, because of the mode softening. A further interesting effect is that the temperature and concentration exponents in (15) and (16), though different in general, become the same for dimension d approaching the lower critical dimension 2. Finally we note that the (unrenormalised) spin wave approach used here does not apply in the limit $y \to 0$ (when M becomes small) and therefore the linear vanishing of M at $T_c(p)$ implied by (15) is expected to be incorrect.

In summary, it has been shown that the crossover from hydrodynamic to critical spin wave dynamics induced by the diverging correlation length near the percolation threshold causes a crossover in a static thermal property, the magnetisation. This crossover is in addition to the usual percolation–thermal crossover, which determines the transition temperature, which also results from the critical dynamics viewpoint. Further details of this work will be presented elsewhere, together with extensions to the antiferromagnetic case and to specific heat, etc. These extensions are non-trivial because of modified zero-point effects and finite cluster contributions respectively.

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References

[1] Aharony A 1985 *Scaling Phenomena in Disordered Systems* ed R Pynn and A Skjeltorp (New York: Plenum) p 289
Orbach R 1985 *Scaling Phenomena in Disordered Systems* ed R Pynn and A Skjeltorp (New York: Plenum) p 335
Stanley H E 1985 *Scaling Phenomena in Disordered Systems* ed R Pynn and A Skjeltorp (New York: Plenum) p 49
[2] Stinchcombe R B 1985 *Scaling Phenomena in Disordered Systems* ed R Pynn and A Skjeltorp (New York: Plenum) p 465
[3] Stinchcombe R B 1983 *Phase Transitions and Critical Phenomena* vol 7, ed C Domb and J L Lebowitz (New York: Academic) p 151
[4] Mandelbrot B 1977 *Fractals: Form, Chance, and Dimension* (San Francisco: Freeman)
[5] Rammal R and Toulouse G 1983 *J. Physique Lett.* **44** L13
[6] Stinchcombe R B 1979 *J. Phys. C: Solid State Phys.* **12** L41
[7] Coniglio A 1981 *Phys. Rev. Lett.* **46** 250
[8] Essam J W 1980 *Rep. Prog. Phys.* **43** 835
[9] Kittel C 1963 *Quantum Theory of Solids* (New York: Wiley)
[10] Halperin B I and Hohenberg P C 1969 *Phys. Rev.* **188** 898
[11] Rammal R 1987 *Time-Dependent Effects in Disordered Materials* ed R Pynn and T Riste (New York: Plenum) to be published
[12] Korenblit I Ya and Shender E F 1978 *Sov. Phys.-Usp.* **21** 832
[13] Harris C K and Stinchcombe R B 1983 *Phys. Rev. Lett.* **50** 1399
[14] Gefen Y, Aharony A and Alexander S 1983 *Phys. Rev. Lett.* **50** 77
[15] Alexander S and Orbach R 1982 *J. Physique Lett.* **43** L625
[16] Grest G S and Webman I 1984 *J. Physique Lett.* **45** L1155